strates a) are accurate to within ± 8 percent, b) are simple to apply, c) are not wasteful of material, and d) can determine the permittivity associated with a specific section of microstrip.

References

[1] B. W. Jervis, R. M. Pannell, and J. A. H. Steeden, "Attenuation in microstrip transmission lines with very lossy substrates", in *1980 IEEE/MTT-S Int. Microwave Symp.* (Washington D.C.), paper V-8, May 1980.

[2] H. A. Wheeler, "Transmission-line properties of parallel strips separated by a dielectric sheet," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 172–185, Mar. 1965.

[3] W. J. Getsinger, "Microstrip dispersion model," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 34–39, Jan. 1973

[4] P. Troughton, "Measurement technique in microstrip," *Electron. Lett.*, vol. 5, No. 2, pp. 25–26, 1969.

[5] I. Wolf and N. Knoppik, "Microstrip ring resonator and dispersion measurement on microstrip lines," *Electron. Lett.*, vol. 7, no. 26, pp. 779–781, 1971.

[6] J. G. Richings, "An accurate experimental method for determining the important properties of microstrip transmission lines," *Marconi Rev.*, fourth quarter, pp. 210–216, 1974.

[7] B. Easter, "The equivalent circuit of some microstrip discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 655–660, Aug. 1975.

[8] J. S. Napoli and J. J. Hughes, "A simple technique for the accurate determination of the microwave dielectric constant for microwave integrated circuits," *IEEE Trans. Microwave Theory Tech*, vol. MTT-19, pp. 664–665, July 1971.

[9] J. Q. Howell, "A quick, accurate method to measure the dielectric constant of microwave integrated circuit substrates," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 142–143, Mar. 1973.

[10] P. H. Ladbrooke, M. H. N. Potok, and E. H. England, "Coupling errors in cavity-resonance measurements on MIC dielectrics," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 560–562, Aug. 1973.

[11] E. O. Hammerstad, "Equations for microstrip circuit design," in Proc. *European Microwave Conf.* pp. 268–272, (Hamburg, Germany), Sept. 1975

Empirical Relations for Capacitive and Inductive Coupling Coefficients of Coupled Microstrip Lines

S. KAL, D. BHATTACHARYA, AND N. B. CHAKRABORTI

***Abstract*—Empirical relations for inductive and capacitive coupling coefficients are proposed. The functional relationships are based on the physical mechanism of coupling in microstrip lines. Values of coupling coefficients computed from even- and odd-mode impedances and phase velocities, available in the literature, are compared with those computed from the proposed relations. Microstrip couplers have been designed on the basis of coupling coefficients to meet the desired coupling and isolation.**

I. Introduction

Microstrip coupled lines are conventionally characterized by their even- and odd-mode characteristic impedances and phase constants [1]–[3]. These may be computed from a knowledge of the even- and odd-mode capacitances [2], [4]. However, there are no simple relations available for finding the parameters involved in the design. An alternative approach to design may be formulated making use of the knowledge of the capacitive and inductive coupling coefficients in coupled lines [5], [6]. Unfortunately, here again straightforward formulas are not available.

This paper aims at finding simple empirical relations describing the variation of both capacitive and inductive coupling coefficients with physical dimensions of the lines and material parameters of the substrates. The relations permit ready application in design of coupled systems realized on dielectric or ferrimagnetic substrates. These should be particularly useful in the optimization process in CAD because of the simple mathematical forms.

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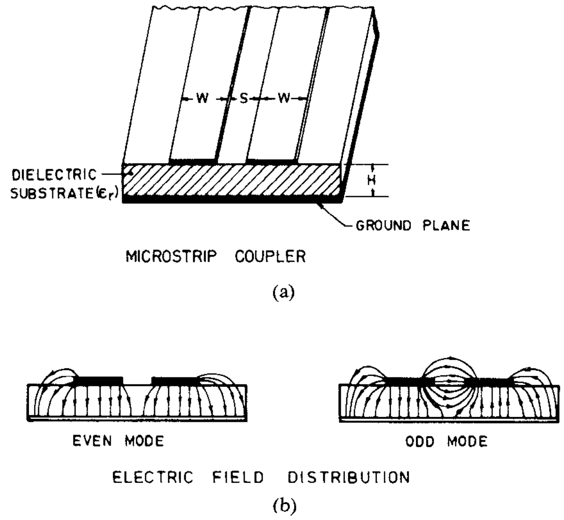

Fig. 1. (a) Schematic diagram of a microstrip coupler. (b) Electric field distribution in even- and odd-mode excitation of the microstrip coupled lines

II. Formulation of the Empirical Relations

A schematic diagram of a coupled line on a substrate characterized by relative permittivity ϵ_r and relative permeability μ_r is shown in Fig. 1(a). The coupling is basically through fringe fields. The extent of the fringe field determines the electric and magnetic coupling coefficients. The field solution of this type of structure outside the region bounded by the top electrode exhibits an exponentially decaying character [7]. The coupling coefficients are thus expected to vary exponentially with the characteristic dimensional ratio (S/H). The results given in Milligan [8] of variation of mutual and self-capacitances can be used to show that their ratio varies exponentially with W/H and S/H. It is also noticed that the variation of capacitive coupling coefficient k_C with S/H is faster than with W/H. The results of Bryant and Weiss [2] indicate that for low values of dielectric constant the variation of coupling with ϵ_r is rapid while it is rather slow for high values of ϵ_r. Characteristic impedances and phase velocities in even and odd modes may be found from a knowledge of even- and odd-mode capacitances and inductances; the inductance is computed from the capacitance of a medium with a relative dielectric constant of unity [4]. This suggests that inductive coupling coefficient k_L should be independent of ϵ_r and should depend on μ_r. Incorporating the above ideas the empirical relations for the coupling coefficients k_C and k_L may now be written as

$$k_C = 0.55 \exp\left[-\left(A_1 S/H + B_1 W/H\right)\right] \tag{1}$$

$$k_L = 0.55 \exp\left[-\left(A_2 S/H + B_2 W/H\right)\right]. \tag{2}$$

A_1, B_1 are functions of ϵ_r and A_2, B_2 are functions of μ_r:

$$A_1(\epsilon_r) = 1 + \frac{1}{4}\ln\left(\frac{\epsilon_r + 1}{2}\right) \qquad B_1(\epsilon_r) = \frac{1}{10}\sqrt{\epsilon_r + 1}$$

$$A_2(\mu_r) = 1 + \frac{1}{4}\ln\left(\frac{\mu_r + 1}{2}\right) \qquad B_2(\mu_r) = \frac{1}{10}\sqrt{\mu_r + 1}. \tag{3}$$

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TABLE I
COMPARISON OF THE VALUES OF k_C AND k_L OBTAINED FROM EMPIRICAL RELATIONS WITH THOSE COMPUTED USING (4) AND (5) FROM THE RESULTS IN REFERENCES CITED

Relative Permittivity of the substrate (ϵ_r)	W/H	S/H	Computed using empirical relations (1) & (2)		Computed using relations (4) & (5)		Results from the references
			k_C	k_L	k_C	k_L	
10 0	0 2	0 2	0 386	0 437	0 386	0 448	[2]
*10 0	0 4	0 2	0 363	0.425	0 367	0.443	
10 0	0 8	0 2	0 317	0 402	0 313	0 409	
10 0	1 0	0 2	0. 296	0 391	0.289	0 392	
10 0	1 5	0 2	0 251	0 364	0 240	0 355	
10 0	2 0	0 2	0 213	0 339	0 206	0 325	
*14 4	0 4	0 2	0 348	0 425	0 359	0 440	[9]
**14 4	0 6	0.4	0 238	0 338	0 252	0.348	
14.4	0.9	0 2	0 286	0.369	0 303	0 408	
14 4	1 1	0 8	0 107	0 211	0 103	0 211	
16 0	0 1	0 2	0 388	0 444	0 370	0 426	[9]
16 0	0 2	0 2	0 373	0.437	0 381	0 447	
16 0	0 4	0 6	0 186	0 284	0 188	0 280	
**16 0	0 6	0 4	0 233	0 338	0 236	0 334	
16 0	1.0	0 6	0 145	0 262	0 141	0 252	
10 4	0 368	0 120	0 408	0 463	0 366	0 474	[3]
10.4	0 785	0 304	0 272	0 363	0 233	0 376	
2 54	2 28	0 05	0 337	0 380	0 303	0 375	[1]
*,** Values of k_C and k_L for same dimensional ratio but different ϵ_r are compared							

TABLE II
PERFORMANCE OF TEST COUPLERS AT THE CENTER FREQUENCY

	CASE A Alumina substrate (ϵ_r=9 6) W/H =0 836 S/H = 0 402			CASE B YIG(Trans Tech G113) substrate (ϵ_r = 15 4) W/H =0 714 S/H =0.620 μ_r =0 387 at H_a =2000 Gauss	
	Design value	More exact* theoretical value	Experimental value	Design value	Experimental value
Coupling in dB	11	10 98	10 30	13	13 30
Isolation in dB	23	23 70	23 50	20	23 50
VSWR	-	-	1 05	-	1. 56

*Expressions used are equations (36)-(38) of reference [6]

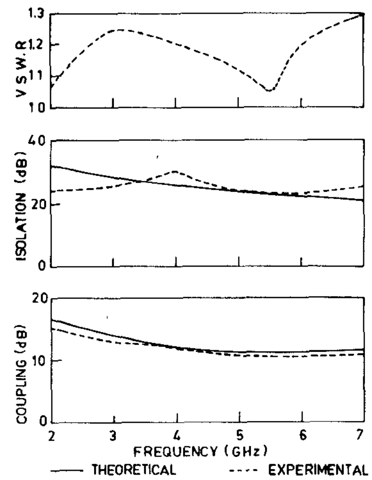

Fig. 2. Measured performance of a test microstrip coupler on alumina substrate (ϵ_r=9.6). The solid and dotted lines indicate theoretical and experimental values, respectively.

The numerical coefficients have been chosen such that the resulting values of k_C and k_L agree with those computed from the even- and odd-mode characteristic impedances (z_{0e} and z_{0o}) and phase velocities (β_{0e} and β_{0o}) quoted by other authors. Values of k_C and k_L can be obtained using relations (22) and (28) of reference [5] and are given by

$$k_C = \frac{z_{0e}\beta_{0o} - z_{0o}\beta_{0e}}{z_{0e}\beta_{0o} + z_{0o}\beta_{0e}} \tag{4}$$

$$k_L = \frac{z_{0e}\beta_{0e} - z_{0o}\beta_{0o}}{z_{0e}\beta_{0e} + z_{0o}\beta_{0o}}. \tag{5}$$

For dielectric substrates μ_r is unity but for magnetic substrates μ_r changes with magnetic field and the variation of k_L with applied magnetic field can be computed using (2) and (3).

III. NUMERICAL RESULTS AND ILLUSTRATIVE DESIGN

Table I presents the values of k_C and k_L computed using (4) and (5) along with those obtained using empirical relations (1) and (2). An asterisk in the Table I is intended to compare the k_C and k_L values for the same dimensional ratio but different ϵ_r. It may be noted that k_L remains substantially constant with variation of ϵ_r while k_C changes with ϵ_r. The empirical relations (1) and (2) are found to be in good agreement (within 5 percent) with available results for $S/H \geqslant 0.05$ and $W/H \geqslant 0.1$ except for the k_C values of last three entries. Unfortunately, for small values of W/H and S/H, results of even- and odd-mode phase velocities are not available and therefore no comparison can be made. The value of coupling corresponding to $S/H=0.05$ and $W/H=0.1$ is 0.534 for alumina substrate ($\epsilon_r=9.6$). For tighter coupling ($S/H<0.05$) accuracy may degrade as then the assumption of exponential variation may not be valid.

A. Illustrative Design

Two designs of microstrip coupler realized on a dielectric and a YIG substrate are discussed in this subsection to illustrate the use of the empirical relations (1) and (2). The performance of a microstrip coupler expressed in terms of coupling, directivity, and VSWR are related to k_C, k_L, and θ_0 (electrical length of the coupled region at center frequency) through the relations given in equations (44)–(46) of reference [6].

1) Case A: For a 25-mil thickness alumina substrate ($\epsilon_r=9.6$), the values of k_C and k_L needed for a 11-dB coupler with 23-dB isolation at a center frequency of 5.5 GHz are 0.237 and 0.327, respectively. Using the empirical relations (1)–(3), the values for W/H and S/H are found to be 0.836 and 0.402, respectively. The W/H ratio of the uncoupled region corresponds to characteristic impedance of 50 Ω. It is observed that the value of characteristic impedance of the coupled region calculated by using the approximate formula $Z_0=\sqrt{z_{0e}z_{0o}}$ is about 52 Ω. The experimentally observed VSWR is found to be less than 1.3 over the frequency range 2–7 GHz. The coupling, isolation, etc., are compared in Table II with the experimentally obtained data at the center frequency.

2) Case B: For a 25-mil thickness YIG (Trans Tech G113) substrate ($\epsilon_r=15.4$, saturation magnetization=1780 G), the values of k_C and k_L needed for a 13-dB coupler with 20-dB isolation at a center frequency of 7.0 GHz are 0.160 and 0.288, respectively. The center frequency has been chosen to be well above f_m [11] which is about 5 GHz for YIG (G113) substrate. The relative permeability (μ_r) is taken to be effective permeability (μ_{eff}) for

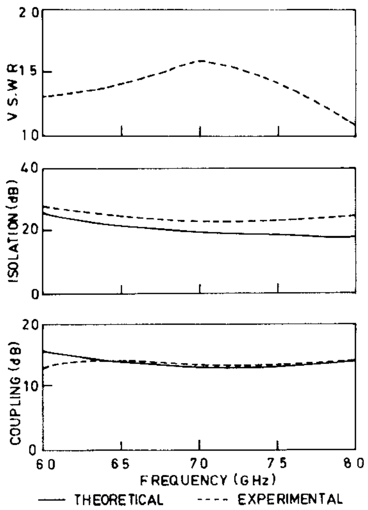

Fig 3. Measured performance of a test microstrip coupler on YIG (Trans Tech. G113) substrate ($\epsilon_r = 15.4$; saturation magnetization = 1780 G; applied magnetic field = 2000 G). The solid and dotted lines indicate theoretical and experimental values, respectively.

magnetization above saturation [1]. As an illustrative case, the magnetic field is chosen to be 2000 G applied transversely to the plane of microstrip lines and for this value of the applied magnetic field μ_{eff} of this substrate at 7 GHz is found to be 0.387 [1], [10]. The values of W/H and S/H for above specifications of the coupler are found to be 0.714 and 0.620, respectively. The experimentally observed values at the centre frequency are compared with the specified values in Table II.

Figs. 2 and 3 give the variations of coupling, isolation, and VSWR of these two couplers over the frequency ranges of investigation and indicate close agreement with design values. The small deviation may be due to etching errors during fabrication of the devices using thin film technique. The empirical relations given here for capacitive and inductive coupling coefficients provide a simple method of designing a microstrip coupler on a dielectric or ferrimagnetic substrate.

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References

[1] L. Young and H. Sobol, *Advances in Microwaves*, vol. 8. New York: Academic, 1974, ch. 5, pp. 203–287, ch. 6, pp. 295–318.

[2] T. G. Bryant and J. A Weiss, "Parameters of microstrip transmission lines and coupled pairs of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 1021–1027, Dec. 1968.

[3] L. S. Napoli and J. J. Hughes, "Characteristics of coupled microstrip lines," *RCA Rev.*, vol. 31, pp. 479–498, Sept. 1970.

[4] J. I. Smith, "The even and odd mode capacitance parameters for coupled lines in suspended substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 424–430, May 1971.

[5] J E. Adair and G. I. Haddad, "Coupled-mode analysis of non-uniform coupled transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 746–752, Oct. 1969

[6] M. K. Krage and G I Haddad, "Characteristics of coupled microstrip transmission lines-I: Coupled mode formulation of inhomogeneous lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 217–222, Apr. 1970.

[7] S. B. Cohn, "Shielded coupled-strip transmission line," *IRE Trans. Microwave Theory Tech*, vol. MTT-3, pp. 29–38, Oct 1955.

[8] T. A. Milligan, "Dimensions of microstrip coupled lines and interdigital structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 405–410, May 1977.

[9] W. M. Libbey, "Theory of active non-reciprocal networks," Ph D. dissertation, Dept. Electrical Eng. Worcester Polytechnic Inst., Worcester, Ma, 1968.

[10] R. A. Pucel and D J. Masse, "Microstrip propagation on magnetic substrates—Part I: Design theory; Part II: Experiment," *IEEE Trans Microwave Theory Tech.*, vol. MTT-20, pp. 304–308 and pp. 309–313, May 1972.

[11] G. R. Harrison *et al.*, "Ferrimagnetic parts of microwave integrated circuits," *IEEE Trans Microwave Theory, Tech.*, vol MTT-19, pp. 577–588, July 1971